

Renormalization in light–cone gauge: how to do it in a consistent way.

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Abstract

We summarize several basic features concerning canonical equal time quantization and renormalization of Yang–Mills theories in light–cone gauge. We describe a “two component” formulation which is reminiscent of the light–cone hamiltonian perturbation rules. Finally we review the derivation of the one–loop Altarelli–Parisi densities, using the correct causal prescription on the “spurious” pole.

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I. INTRODUCTION

Axial type gauges, characterized by the homogeneous $n^\mu A_\mu = 0$ or inhomogeneous $n^\mu A_\mu = \phi$ conditions, n^μ being a fixed constant vector and ϕ a free field, have been considered long time ago, in particular since the beginning of perturbative QCD calculations [1].

They are often called “physical” or “unitary” gauges, although this is not completely true, as it will appear in the sequel. Certainly, they trade manifest Lorentz covariance in favour of the absence of unphysical degrees of freedom, at least in the homogeneous case [2]. For this reason they are particularly suitable in perturbative calculations: planar diagrams dominate in deep-inelastic scattering and are endowed with a transparent partonic interpretation.

In supersymmetric theories, the light-cone gauge ($n^2 = 0$) enjoys the property of having equal number of “transverse” independent fields and of “physical” excitations. Finiteness of SUSY $N = 4$ can thereby most naturally be proven [3].

A further simplification occurs owing to the decoupling of the Faddeev–Popov determinant, at least for trivial topological configurations.

Still one has to bear in mind that they are “singular” gauges: delicate prescriptions are in order when handling Feynman propagators in perturbative calculations. Even more delicate is the issue concerning the possibility of regularizing and eventually renormalizing Green’s functions.

This is the main topic discussed in the sequel: we shall use dimensional regularization throughout. The goal of bringing algebraic non covariant gauges to a level of accuracy comparable to the one obtained in the more familiar Feynman gauge has been achieved [2] and is now a matter for textbooks.

Lorentz covariance is recovered in these gauges by the combined use of the Dirac formulation of constrained systems together with a weak condition, when necessary, to single the “physical” Hilbert space out of an indefinite metric Fock space. Lorentz covariance is achieved once all observable quantities possess correct transformation properties under the

Poincaré algebra, possibly restricted to the “physical” subspace. This is exactly what the equivalence principle requires and has been carefully discussed in ref. [2].

Two gauge choices showed up to be viable so far, although on a quite different status: the spacelike planar gauge $n^\mu A_\mu = \phi, n^2 < 0$ and the light-cone gauge $n^\mu A_\mu = 0, n^2 = 0$. They share the following form of the free Feynman propagator

$$D_{\mu\nu}(k) = \frac{i}{k^2 + i\epsilon} [-g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{nk}]. \quad (1)$$

The quantity $(nk)^{-1}$ needs a prescription in order to represent a well defined distribution. We shall first briefly comment the spacelike case and then focus our attention on the light-cone gauge.

II. THE SPACELIKE OPTION

When $n^2 < 0$, one can choose $n_\mu = (0, 0, 0, 1)$ without loss of generality. Then the singularity $(nk)^{-1}$ does not interfere with the causal Feynman poles at $k^2 = 0$; in particular the integration contour can be Wick rotated without extra terms.

Canonical quantization suggests the Cauchy principal value (P) for $(nk)^{-1}$ in this case [4].

The field ϕ has the wrong sign in its quantum algebra, namely it behaves like a “ghost”. Nevertheless, being a free field, it can be consistently excluded from the “physical” Hilbert space by means of the weak condition $\phi^{(-)}|\Phi_{\text{phys}}\rangle = 0$.

However ambiguities arise in higher orders: the only mathematically sound way to interpret $(nk)^{-2}$ is in the distribution sense

$$(nk)^{-2} \equiv -\frac{d}{d(nk)} P\left(\frac{1}{nk}\right), \quad (2)$$

which spoils positivity; as a consequence consistency with unitarity is not granted. The algebraic splitting formula

$$P\left(\frac{1}{nk}\right)P\left(\frac{1}{n(p-k)}\right) = \frac{1}{np} [P\left(\frac{1}{nk}\right) +$$

$$+ P(\frac{1}{n(p-k)})] \quad (3)$$

does not hold . As a matter of fact of fact the Poincaré-Bertrand term

$$- \pi^2 \delta(nk) \delta(np) \quad (4)$$

should be added, which has always been disregarded in practical calculations. In order to justify this procedure, loop integrals require the use of peculiar functional spaces (Besov spaces) as well as delicate considerations concerning the adiabatic switching of the interaction. This has been discussed at length in [5], where exponentiation of the Wilson loop up to the order g^4 has been proven as a test of gauge invariance. However, beyond this perturbative order, there is no guarantee of consistency; the renormalization proposed in [6] has thereby to be regarded only in a formal sense.

Finally the limit $n^2 \rightarrow 0$ turns out to be singular and generally out of control when setting up renormalization.

III. THE LIGHT-CONE CHOICE

For all the previous reasons it is worth considering the light-cone gauge $n^u A_\mu = 0, n^2 = 0$, to be imposed in a strong sense, i.e. by means of a Lagrange multiplier λ . It is not restrictive to choose $n_\mu = \frac{1}{\sqrt{2}}(1, 0, 0, 1)$. One easily recovers the expression (1) for the free Feynman propagator, but now the singularity at $nk = 0$ can interfere with the Feynman poles at $k^2 = 0$.

If P-prescription (or a sharp infrared cutoff) is adopted in analogy with the spacelike case, causality (and thereby analyticity) is violated. As a matter of fact the Cauchy principal value distribution is always the sum of a causal pole and of an anti-causal one. The latter produces an extra unwanted term under Wick rotation through a pinch of the integration contour.

Power counting control of superficially divergent Feynman diagrams is lost together with all standard theorems (Weinberg – BPHZ) which stand at the very basis of renormalization

[7].

A mismatch occurs between ultraviolet and collinear singularities; renormalization *constants* turn out to be momentum dependent. In these conditions, although correct results for higher order contributions in particular instances cannot be excluded a priori if clever recipes are followed, they are not supported by any sound general procedure.

Equal time canonical quantization induces a causal behaviour on the singularity $nk = 0$ [8]:

$$\begin{aligned} \frac{1}{k_0 - k_3} &\equiv \frac{1}{k_0 - k_3 + i \in \text{sign}(k_0 + k_3)} = \\ &= \frac{k_0 + k_3}{k_0^2 - k_3^2 + i \in} \end{aligned} \quad (5)$$

which, in turn, allows a Wick rotation without extra terms.

The first form of eq.(5) was heuristically proposed by Mandelstam [9], the second one by Leibbrandt [10] (ML prescription).

The free propagator now possesses two absorptive parts [11]

$$\begin{aligned} \text{disc} D_{\mu\nu}(k) &= 2\pi\theta(k_0)\delta(k^2) \cdot \\ &\cdot \left[-g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{k_\perp^2} \frac{2\hat{n}k}{n\hat{n}} \right] - 2\pi\theta(k_0) \cdot \\ &\cdot \delta(k^2 + k_\perp^2) \frac{2\hat{n}k}{n\hat{n}} \frac{n_\mu k_\nu + n_\nu k_\mu}{k_\perp^2}, \end{aligned} \quad (6)$$

where $\hat{n}_\mu = \frac{1}{\sqrt{2}}(1, 0, 0, -1)$.

The second contribution has the wrong sign, namely it is of a “ghost” type. We stress that it is not an optional choice, it is an unavoidable consequence of equal time canonical quantization. Its presence naturally protects the collinear behaviour ($k_\perp = 0$) of the propagator.

Negative norm states occur in the perturbative Fock space; however they are consistently expunged from the “physical” Hilbert space by imposing Gauss’ law in a “weak” sense [8]. In this Hilbert space unitarity is automatically restored.

The possibility of a Wick rotation without extra terms leads to power counting control of superficially divergent graphs. Standard theorems are recovered, provided two separate

countings are performed with respect to a dilatation of all momentum components and of only “transverse” ones. Convergence requires both indices to be negative.

Ultraviolet and infrared singularities become fully disentangled: Green functions in euclidean regions of momenta exhibit only ultraviolet singularities which appear as poles at $D = 4$, D being the number of dimensions.

One particle irreducible vertices turn out to have poles at $D = 4$ with residues which sometimes involve non polynomialities with respect to “external” momenta of the type $(np)^{-1}$. Non local counterterms are thereby required, although of a very special kind [12].

After a careful study of all possible tensorial structures, after imposing Ward identities, which are simple in light-cone gauge, and further technical conditions needed to match with the spacelike case, in ref. [12] it has been shown that there is only one non local acceptable structure

$$\Omega = (nD)^{-1} \frac{n^\mu F_{\mu\nu} \hat{n}^\nu}{n\hat{n}}, \quad (7)$$

$F_{\mu\nu}$ being the usual field tensor and D the covariant derivative acting on the adjoint representation; $(nD)^{-1}$ is to be understood in a perturbative sense, with causal boundary conditions.

Ω is a covariant quantity with mass dimension equal to unity. It gives rise to the counterterm in the effective action

$$\Delta = \Omega n^\mu [D^\nu F_{\mu\nu} - g \bar{\psi} \tau \gamma_\mu \psi], \quad (8)$$

where one recognizes the classical equation of motion, as expected on general grounds [13].

The canonical transformation

$$\begin{aligned} A_\mu^{(0)} &= Z_3^{1/2} [A_\mu - (1 - \tilde{Z}_3^{-1}) n_\mu \Omega], \\ \psi^{(0)} &= Z_2^{1/2} \left(\frac{\not{n} \not{\psi}}{2n\hat{n}} + \tilde{Z}_2 \frac{\not{n} \not{\psi}}{2n\hat{n}} \right) \psi, \\ g_0 &= Z_3^{-1/2} g, \\ \lambda^{(0)} &= Z_3^{-1/2} \lambda \end{aligned} \quad (9)$$

relates bare and renormalized fields through the appearance of four renormalization constants. Only two of them (Z_2, Z_3) are however independent, as it will be explained in the sequel.

All Green functions have been explicitly computed at one loop, in particular the renormalization constants at $\mathcal{O}(g^2)$ [2]. Results at two loop level have also been obtained. The correct exponentiation of a Wilson loop with light-like sides has been checked $\mathcal{O}(g^4)$ together with a calculation of the related anomalous dimensions at the same order [14].

One should also mention an interesting result concerning composite operators: it has been shown at any order in the loop expansion [15] that gauge invariant composite operators in light-cone gauge mix under renormalization only among themselves, at variance with their behaviour in covariant gauges [16].

IV. THE TWO-COMPONENT FORMULATION

We would like to discuss a “two component” formulation which may be useful in particular instances. Let’s start from the Green function generating functional

$$W[J, \eta] = \int d[A_\mu, \lambda, \bar{\psi}, \psi] e^{i \int d^4x [\mathcal{L} + \mathcal{L}_s]}, \quad (10)$$

where

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \lambda n A + \bar{\psi} (i \not{D} - m) \psi, \\ \mathcal{L}_s &= J_\mu A^\mu + \bar{\eta} \psi + \bar{\psi} \eta. \end{aligned} \quad (11)$$

J and η are external sources, λ a Lagrange multiplier enforcing the condition $nA = A_- = 0$; colour indices are understood. Let us also introduce the projection operators

$$P_+ = \frac{\not{\eta} \not{\eta}}{2n\hat{n}}, \quad P_- = \frac{\not{\eta} \not{\eta}}{2n\hat{n}}, \quad P_+ + P_- = 1. \quad (12)$$

In light-cone gauge W is gaussian with respect to the variables A_+ and χ :

$$\chi = P_- \psi, \quad \varphi = P_+ \psi, \quad \psi = \varphi + \chi. \quad (13)$$

Similarly we define

$$\xi = P_+ \eta, \quad \zeta = P_- \eta. \quad (14)$$

Then, integrating over A_+ and χ , we get

$$W = \exp\left[\frac{i}{2} \int (J^+ \partial_-^2 J^+ + \bar{\xi} \frac{i\gamma^+ \partial_+}{\partial_+ \partial_-} \xi) d^4 x\right] \cdot \int d[A_\alpha, \varphi, \bar{\varphi}] e^{i \int (\mathcal{L}_{eff} + \mathcal{L}_{mix} + \hat{\mathcal{L}}_s) d^4 x}, \quad (15)$$

where

$$\begin{aligned} \mathcal{L}_{eff} &= -\frac{1}{4} F_{\alpha\beta} F_{\alpha\beta} + \partial_+ A_\alpha \partial_- A_\alpha + \\ &\quad + i\bar{\varphi} \gamma^+ \partial_+ \varphi - \frac{1}{2} K^2 + \frac{1}{2} \bar{\varphi} (i\gamma^\alpha \mathcal{D}_\alpha - \\ &\quad - m) \frac{i\gamma^+ \partial_+}{\partial_+ \partial_-} (i\gamma^\alpha \mathcal{D}_\alpha - m) \varphi, \\ \mathcal{L}_{mix} &= -K \partial_-^{-1} J^+ + \frac{1}{2} \bar{\xi} \frac{i\gamma^+ \partial_+}{\partial_+ \partial_-} (i\gamma^\alpha \mathcal{D}_\alpha - \\ &\quad - m) \varphi + \frac{1}{2} \bar{\varphi} (i\gamma^\alpha \mathcal{D}_\alpha - m) \frac{i\gamma^+ \partial_+}{\partial_+ \partial_-} \xi, \\ \hat{\mathcal{L}}_s &= J^\alpha A_\alpha + \bar{\zeta} \varphi + \bar{\varphi} \zeta. \end{aligned} \quad (16)$$

In eqs.(16) $\alpha = 1, 2$ and

$$K = \partial_-^{-1} [D_\alpha \partial_- A_\alpha + g \bar{\varphi} \gamma^+ T \varphi]. \quad (17)$$

Moreover ∂_-^{-1} and $(\partial_+ \partial_-)^{-1}$ have always to be understood with causal boundary conditions.

Only “transverse” fields, A_α and φ , appear in eq.(15); the dependent fields A_+ and χ can be expressed in terms of A_α and φ , although in a non local way. Their Green functions can also be expressed by means of “bridge identities” (BI) [11] in terms of the independent “transverse” ones: in particular the renormalization constants \tilde{Z}_2 and \tilde{Z}_3 can be obtained as dependent quantities at any order in the loop expansion.

The (BI) read

$$\begin{aligned}
\partial_-^2 \frac{\delta}{i\delta J^+} &= J^+ + D_\alpha \left[\frac{\delta}{i\delta J_\alpha} \right] \partial_- \frac{\delta}{i\delta J_\alpha} + \\
&+ g \frac{\delta}{i\delta \zeta} \gamma^+ T \frac{\delta}{i\delta \bar{\zeta}}, \\
2\partial_- \frac{\delta}{i\delta \xi} &= i\gamma^+ \xi + i\gamma^+ \cdot \\
&\cdot [i\gamma^\alpha \mathcal{D}_\alpha \left[\frac{\delta}{i\delta J_\alpha} \right] - m] \frac{\delta}{i\delta \bar{\zeta}}, \\
2\partial_- \frac{\delta}{i\delta \xi} &= i\bar{\xi} \gamma^+ + \frac{\delta}{i\delta \zeta} \cdot \\
&\cdot [i\gamma^\alpha \mathcal{D}_\alpha \left[\frac{\delta}{i\delta J_\alpha} \right] - m] i\gamma^+,
\end{aligned} \tag{18}$$

where the operators are supposed to act on W of eq.(15). These identities hold to any order in perturbation theory and usually mix terms with different powers of the coupling constant g .

If only transverse Green functions are sought, one can set J^+ and ξ equal to zero in eq.(15).

In the “two component” formulation new vertices appear with non polynomial character and complicate topology, already at the tree level. They bear no simple relation with the vertices of the “four component” formulation. They are reminiscent of the vertices occurring in light-cone hamiltonian theory [18]. However the ML prescription prevents from integrating first over the $(+)$ -momentum components; a transition to the old-fashioned perturbation theory is thereby impossible, unless peculiar subtractions are performed “step-by-step” [17].

Renormalization cannot be directly proven in the “two component” formulation, because the basic theorems do not apply. However, from the transformation (9), one can easily obtain

$$W[J_\alpha, \zeta] = \int d[A_\alpha, \varphi, \bar{\varphi}] e^{i \int (\mathcal{L}_R + \mathcal{L}_s) d^4x}, \tag{19}$$

where

$$\mathcal{L}_R = -\frac{Z_3}{4} F_{\alpha\beta} F_{\alpha\beta} + Z_3 \partial_+ A_\alpha \partial_- A_\alpha +$$

$$\begin{aligned}
& + iZ_2\bar{\varphi}\gamma^+\partial_+\varphi - \frac{Z_3}{2}(\partial_-^{-1}[D_\alpha\partial_-A_\alpha + \\
& + g\frac{Z_2}{Z_3}\bar{\varphi}\gamma^+T\varphi])^2 + \frac{Z_2}{2}[\bar{\varphi}(i\gamma^\alpha\mathcal{D}_\alpha - m) \cdot \\
& \cdot \frac{i\gamma^+\partial_+}{\partial_+\partial_-}(i\gamma^\alpha\mathcal{D}_\alpha - m)\varphi].
\end{aligned} \tag{20}$$

“Unphysical” renormalization constants \tilde{Z}_2 and \tilde{Z}_3 no longer occur, nor the non local quantity Ω . They are buried in the non local structures which are produced when developing perturbation theory starting from the functional (19).

V. THE ALTARELLI–PARISI DENSITIES

One loop Altarelli–Parisi (AP) splitting functions have been correctly recovered in this causal light–cone formulation [19]; the basic new feature is the appearance of the well–defined $(1-x)_+^{-1}$ distribution already in the “real” contributions.

Let us briefly review this derivation.

Kinematics can be usefully parametrized as

$$\begin{aligned}
p_\mu &= (P + \frac{p^2}{4P}, \underline{0}, P - \frac{p^2}{4P}), \\
k_\mu &= (\xi P + \frac{k^2 + k_\perp^2}{4\xi P}, \underline{k}_\perp, \xi P - \frac{k^2 + k_\perp^2}{4\xi P}),
\end{aligned} \tag{21}$$

and

$$n_\mu = (\frac{np}{2P}, \underline{0}, -\frac{np}{2P}), \quad \hat{n}_\mu = (\frac{P}{np}, \underline{0}, \frac{P}{np}). \tag{22}$$

Here ξ represents the fraction of the (large) longitudinal component P of the incoming quark momentum p (small $p^2 < 0$), carried by k .

The AP density is the coefficient of the term $\log |\frac{Q^2}{p^2}|$, in the propagation kernel, when $p^2 \rightarrow 0$, $-Q^2$ being the (large) virtuality of the external current up to which the vector k has to be integrated. To the “real” part of the kernel $K^{(a)}$ we associate the quantity [20]

$$K_q^{(a)}(x, |\frac{Q^2}{p^2}|) = g^2 c_F \int \frac{d^4 k}{(2\pi)^4}.$$

$$\begin{aligned}
& \cdot \delta(1 - \frac{nk}{xnp}) \frac{1}{(k^2)^2} \cdot \text{Tr}[\frac{\not{n}}{4nk} \not{k} \gamma^\mu \not{p} \gamma^\nu \not{k}] \cdot \\
& \cdot \text{disc}[D_{\mu\nu}(p-k)],
\end{aligned} \tag{23}$$

the discontinuity being the one of eq.(6). The spin trace is self-explanatory but the factor $\frac{\not{n}}{4nk}$, which is introduced to project the “leading-log” contribution; c_F is the usual colour factor. We can safely work in four dimensions, as no ultraviolet (UV) singularities occur since we are evaluating an absorptive part and no infrared (IR) singularities either, as long as $p^2 < 0$, thanks to the ML prescription. A straightforward calculation gives

$$\begin{aligned}
K_q^{(a)} &= \frac{g^2 c_F}{8\pi^2} \int_{-p^2}^{-Q^2} \frac{d|k^2|}{|k^2|} \int d(k_\perp^2) \cdot \\
& \cdot \delta(1 - x - \frac{k_\perp^2}{|k^2|}) [\frac{1-x}{|k^2|} + \frac{2x}{k_\perp^2}] - \\
& - \frac{g^2 c_F}{8\pi^2} \int_{-p^2}^{-Q^2} \frac{d|k^2|}{|k^2|} \int d(k_\perp^2) \frac{2}{k_\perp^2} (1 - \frac{k_\perp^2}{|k^2|}) \cdot \\
& \cdot \theta(|k^2| - k_\perp^2) \delta\left((1-x)(1 - \frac{k_\perp^2}{|k^2|})\right),
\end{aligned} \tag{24}$$

the second addendum arising from the presence of the ghost. Both contributions are singular at $x = 1$, but they nicely combine; we have indeed

$$\begin{aligned}
K_q^{(a)} &= \frac{g^2 c_F}{8\pi^2} \log \left| \frac{Q^2}{p^2} \right| \left[(1-x + \frac{2x}{1-x}) - \right. \\
& \left. - 2\delta(1-x) \int_0^1 \frac{dy}{1-y} \right],
\end{aligned} \tag{25}$$

namely the well defined distribution

$$K_q^{(a)} = \frac{g^2 c_F}{8\pi^2} \log \left| \frac{Q^2}{p^2} \right| [-1 - x + \frac{2}{(1-x)_+}]. \tag{26}$$

We notice that the IR singularity at $x = 1$ is fully regularized by the ghost, already in the diagram describing the “real” contribution.

The one-loop self-energy, regularized in $D = 2w$ dimensions, has been discussed at length in ref. [2]. It has the expression (μ is here the renormalization scale)

$$\Sigma(p) = -\frac{ig^2 c_F}{16\pi^2} (\frac{-p^2}{4\pi\mu^2})^{w-2} \left[-\frac{\not{p}}{\sin(\pi w)} \right].$$

$$\begin{aligned}
& \cdot \frac{\pi\Gamma(w)}{\Gamma(2w-2)} + 2\frac{\not{n}\hat{n}p}{n\hat{n}} \left[\left(1 + \frac{\Gamma(w)\Gamma(w-1)}{\Gamma(2w-2)}\right) \cdot \right. \\
& \left. \cdot \Gamma(1-w) + \frac{\pi^2}{6} + \psi'(2) \right] \Bigg]. \tag{27}
\end{aligned}$$

It exhibits the nice feature of having no IR singularities as long as $p^2 < 0$, at variance with expressions obtained in previous treatments, in which P-prescription was adopted.

From eq.(27) one easily realizes that the one loop radiative correction at the pole $p^2 = 0$ of the fermion propagator renormalized in the minimal subtraction scheme is

$$\Delta S^R = -\frac{3g^2c_F}{16\pi^2} \log\left(\frac{-p^2}{4\pi\mu^2}\right) + f.t. \tag{28}$$

where *f.t.* refers to terms which are finite in the limit $p^2 \rightarrow 0$.

This result, together with eq.(26), finally gives

$$\begin{aligned}
K_q(x, p^2) &= \frac{g^2c_F}{8\pi^2} \log \left| \frac{p^2}{\mu^2} \right| [1 + x - \\
&- \frac{2}{(1-x)_+} - \frac{3}{2}\delta(1-x)] + f.t. \tag{29}
\end{aligned}$$

and one recognizes the flavour non singlet AP density

$$\begin{aligned}
P_q^q(x) &= \frac{\alpha_s c_F}{2\pi} \left[-1 - x + \frac{2}{(1-x)_+} + \right. \\
&+ \left. \frac{3}{2}\delta(1-x) \right] \equiv \frac{\alpha_s c_F}{2\pi} \left(\frac{1+x^2}{1-x} \right)_+, \tag{30}
\end{aligned}$$

with $\alpha_s = g^2/4\pi$.

We notice that, were we interested in computing branching probabilities, both the ghost and the virtual radiative corrections at the fermion pole should be omitted, and the IR singularity at $x = 1$ would be fully exposed

$$\hat{P}_q^q = \frac{\alpha_s c_F}{2\pi} \frac{1+x^2}{1-x}. \tag{31}$$

Should we instead be interested in Sudakov form factor, the gluon radiation (but not the ghost one!) should be inhibited in the absorptive part and the usual result would be easily recovered.

One-loop unitarity sum rules relate real $r(x)$ and virtual $v(x)$ contributions, as is well known [21]. In our approach both quantities are separately well defined, as anytime a gluon is summed over, the ghost is standing by it [11], to protect its IR behaviour with the appropriate δ -measure.

As a matter of fact, in the flavour non-singlet case, we have

$$\begin{aligned}
v_q^q(x) &= -\frac{1}{2}\delta(1-x) \int_0^1 dy [r_q^q(y) + r_q^g(y)] = \\
&= -\frac{\alpha_s C_F}{4\pi} \delta(1-x) \int_0^1 dy \left([-1-y+ \right. \\
&\quad \left. + \frac{2}{(1-y)_+}] + [-2+y+\frac{2}{y_+}] \right) = \\
&= \frac{3\alpha_s C_F}{4\pi} \delta(1-x),
\end{aligned} \tag{32}$$

and thereby

$$P_q^q(x) = r_q^q(x) + v_q^q(x), \tag{33}$$

as expected. We stress that $v_q^q(x)$ is positive, at variance with previous treatments, owing to the ghost contribution. In turn the real contribution is negative due to its “overshielding”. In spite of those paradoxical behaviours, they nicely combine to give the correct answer for any quantity of physical interest.

Now we repeat the calculation for the gluon-gluon case.

We introduce the vectors

$$e_\mu^{(\beta)}(k) = -g_{\mu\beta} + \frac{n_\mu k_\beta}{[nk]}, \quad \beta = 1, 2. \tag{34}$$

These vectors enjoy the property of being orthogonal to both n_μ and k_μ . We have indeed

$$n^\mu e_\mu^{(\alpha)} = k^\mu e_\mu^{(\alpha)} = 0. \tag{35}$$

When $k^2 = 0$, $nk = \frac{k_\perp^2}{2\hat{n}+k_-}$ and $e_\mu^{(\alpha)}$ become the two (physical) polarization vectors.

If we are interested in structure function, the vector $q = p - k$ is on-shell as we are computing just an absorptive part, the vector p is slightly off-shell and the vector k is

spacelike. As ξ cannot vanish in the kinematical region of interest, the prescription in eq.(5) is irrelevant both for the vectors $e_\mu^{(\alpha)}(p)$ and $e_\rho^{(\beta)}(k)$. One can also show that

$$\begin{aligned} \sum_{\alpha=1}^2 e_\mu^{(\alpha)}(k) e_\nu^{(\alpha)}(k) &= -d_{\mu\rho}(k) d_\nu^\rho(k) = \\ &= d_{\mu\nu}(k) - \frac{n_\mu n_\nu k^2}{[nk]^2}. \end{aligned} \quad (36)$$

Then we define the tensor

$$\begin{aligned} T^{\nu\nu'} &= \frac{1}{2} \sum_{\alpha,\beta} \left[e_\mu^{(\alpha)}(p) e_\rho^{(\beta)}(k) V^{\mu\rho\nu} \right] \\ &\cdot \left[e_{\mu'}^{(\alpha)}(p) e_{\rho'}^{(\beta)}(k) V^{\mu'\rho'\nu'} \right], \end{aligned} \quad (37)$$

$V^{\mu\rho\nu}$ being the triple gluon vertex (we have here averaged over initial polarizations).

The usual definition of the gluon–gluon kernel entails the quantity

$$\begin{aligned} K_g^{(a)} &= ig^2 c_A \int \frac{d^4 k}{(2\pi)^4} \delta\left(1 - \frac{nk}{xnp}\right) \cdot \\ &\cdot \frac{T^{\mu\nu}}{(k^2)^2} \text{disc}[D_{\mu\nu}(p-k)], \end{aligned} \quad (38)$$

c_A being the relevant colour factor.

A lengthy but straightforward calculation gives for $x > 0$

$$\begin{aligned} K_g^{(a)} &= \frac{g^2 c_A}{4\pi^2} \log \left| \frac{Q^2}{p^2} \right| \left[x(1-x) + \right. \\ &+ \frac{1-x}{x} + \frac{x}{1-x} - \delta(1-x) \int_0^1 \frac{dy}{1-y} \Big] \equiv \\ &\equiv \frac{g^2 c_A}{4\pi^2} \log \left| \frac{Q^2}{p^2} \right| \cdot \left[x(1-x) + \right. \\ &+ \frac{1-x}{x} - 1 + \frac{1}{(1-x)_+} \Big]. \end{aligned} \quad (39)$$

The ghost is responsible for the term with the δ -function. Again both contributions are singular at $x = 1$, but the IR singularity at $x = 1$ is exactly regularized already in this real kernel.

The one loop expression of the self-energy tensor has been completely evaluated [22]. It will not be reported here. We give instead the one loop radiative corrections to the

transverse components of the vector propagator for pure Yang–Mills theory, renormalized in the minimal subtraction scheme

$$\Delta D_{\alpha\beta}^R = -\frac{ig_{\alpha\beta}}{p^2} \frac{g^2 c_A}{16\pi^2} \frac{11}{3} \log\left(\frac{4\pi\mu^2}{-p^2}\right) + f.t. \quad (40)$$

where f.t. refer again to terms which are finite in the limit $p^2 \rightarrow 0$.

Collecting the one-loop virtual radiative corrections at the gluon pole with the expression (39) we get for $x > 0$

$$K_g = \frac{g^2 c_A}{4\pi^2} \log\left(\frac{4\pi\mu^2}{-p^2}\right) \left[x(1-x) + \frac{1-x}{x} - 1 + \frac{1}{(1-x)_+} + \frac{11}{12} \delta(1-x) \right] + f.t., \quad (41)$$

leading to the corresponding well known AP density

$$P_g^g = \frac{\alpha_s}{2\pi} 2c_A \left[\frac{1-x}{x} + \left(\frac{x}{1-x}\right)_+ + x(1-x) - \frac{1}{12} \delta(1-x) \right]. \quad (42)$$

To (42) one should add the quark contribution we have disregarded in (40), giving the extra term $-\frac{\alpha_s}{6\pi} n_F \delta(1-x)$ (n_F being the flavour number); it does not entail any difference with respect to previous treatments. Again, when computing the probability density for real gluon emission, ghost contribution and virtual corrections should be omitted, thereby recovering full symmetry under the exchange $x \leftrightarrow 1-x$.

Finally, in the gluon case we can check again the unitarity sum rule

$$\begin{aligned} v_g^g(x) &= -\frac{1}{2} \delta(1-x) \int_0^1 dy [r_g^g(y) + 2r_g^q(y)] = \\ &= -\frac{\alpha_s}{2\pi} \delta(1-x) \int_0^1 dy \left(c_A [y(1-y) - 2 + \frac{1}{(1-y)_+} + \frac{1}{y_+}] + \frac{n_F}{2} [y^2 + (1-y)^2] \right) = \\ &= \frac{\alpha_s}{2\pi} \delta(1-x) \left[\frac{11}{6} c_A - \frac{1}{3} n_F \right]. \end{aligned} \quad (43)$$

As a concluding remark, we hope that the successful calculation we have just reported may encourage people to apply the procedure we have described above, in a systematic way to higher order calculations.

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